

## Traveltime tomography inversion

### Using:

First arrival refraction tomography using FAST

Joint refraction and single reflector tomography using TOMO2D

[louise.watremez@dal.ca](mailto:louise.watremez@dal.ca)

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## Traveltime tomography inversion

Preparation for the modeling

Modeling the seismic arrival times with FAST and Tomo2D

Quantitative and qualitative resolutions of the preferred model, or "How to present your tomography results?"

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## Preparation for the modeling

Data picking

Estimation of the picking uncertainties

Initial velocity model

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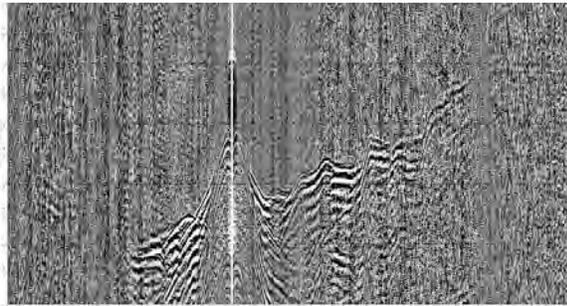
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**Preparation for the modeling - Picking**

Example of data



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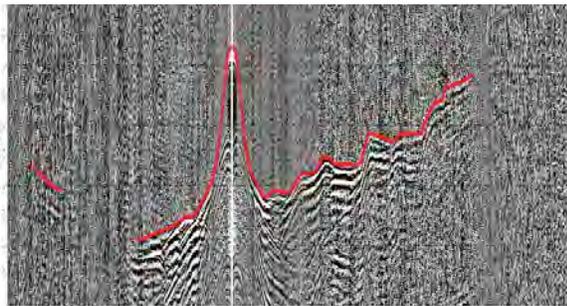
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**Preparation for the modeling - Picking**

Picking of the first arrivals (red)



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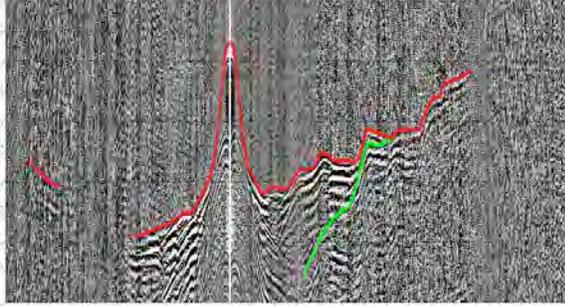
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### Preparation for the modeling - Picking

Picking of the first arrivals (red)  
and Moho wide-angle reflections (green)



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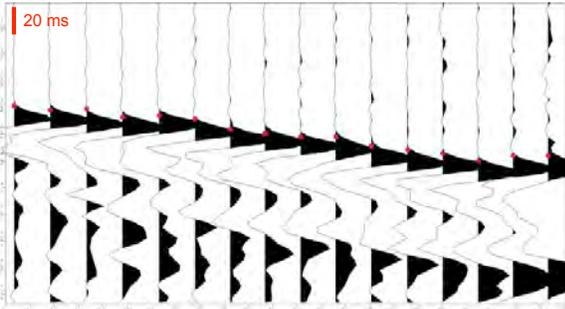
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### Preparation for the modeling - Uncertainties

Small offsets (2-4 km) - signal to noise ratio very low  
Minimum picking uncertainty



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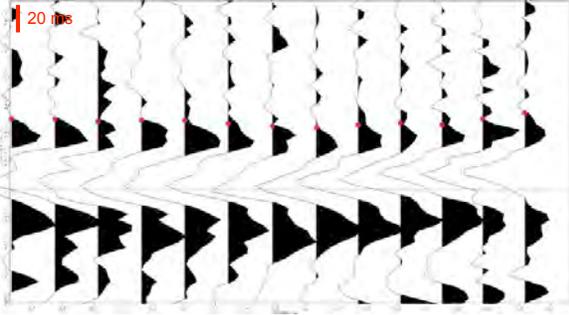
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### Preparation for the modeling - Uncertainties

>10 km offsets - signal to noise ratio a little bit higher  
Still low picking uncertainty



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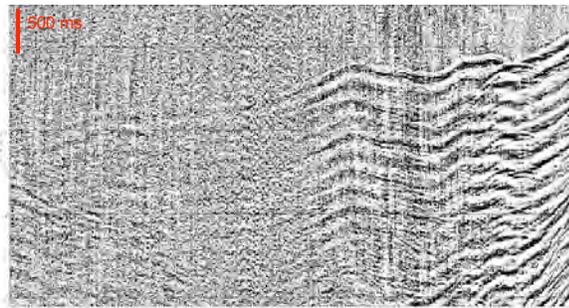
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### Preparation for the modeling - Uncertainties

Variation of the signal to noise ratio with the offset  
Need to adapt the picking uncertainty



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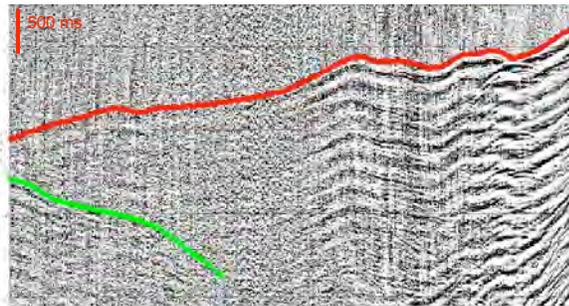
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### Preparation for the modeling - Uncertainties

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Need to adapt the picking uncertainty



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### Preparation for the modeling - Uncertainties

The signal to noise ratio of the data around a pick varies depending on the offset  
It also varies with the instrument, the seafloor conditions (tidal noise, currents, geology...).

Different solutions to assign uncertainty values to your picks:

- "by hand"
- offset dependent ( $unc = a \cdot offset + b$ )
- signal to noise ratio dependent (see Zelt and Forsyth, 1994)

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### Preparation for the modeling

Data picking  
Estimation of the picking uncertainties  
**Initial velocity model**

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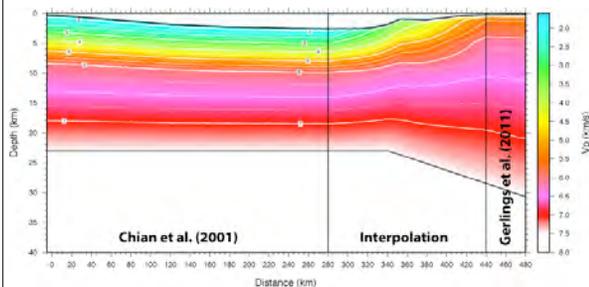
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### Preparation for the modeling - Initial velocity model



As simple as possible  
If possible: no velocity jumps  
Use available a-priori information

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### Modeling with FAST (Zelt and Barton, 1998)

Model parameterization:

- 2D velocity model parameterized as a uniform grid; Sources and receivers anywhere in the model; Different grids for forward and inverse problems

Forward problem:

- Traveltimes and raypaths calculated by solving the eikonal equation by finite differencing

Inverse problem:

- Iterative application of a sparse least squares regularized inversion (LSQR variant of conjugate gradient method); Top part of the model can be "frozen" during inversion

Goal:

- Minimum structure velocity model that satisfies chosen normalized chi-square test (model includes only structure required to fit the data according to its noise level)

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### FAST - Model parameterization

Grid size of the file with the model = grid size for the forward modeling.

The grid size for the forward modeling controls:

- The accuracy of the synthetic traveltimes and ray paths
- The computing time for the forward problem
- Need to find a good balance

The grid size for the inverse model controls:

- The resolution
- The number of unknowns (velocities in cells) and thus, the stability of the model
- Need to find a good balance too

**PARAMETRIC STUDIES !**

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## FAST - Forward problem

### 2 parameter files:

f.in and r.in to control the finite differences computing and calculation of the ray paths.

f.in: parameters for FD, position of sources (OBSs for marine and shots for land experiments)

r.in: parameters for ray paths computation

FD will look for the shortest traveltimes across the grid between the source and the receiver.

RAY will compute the ray paths with the output of FD.

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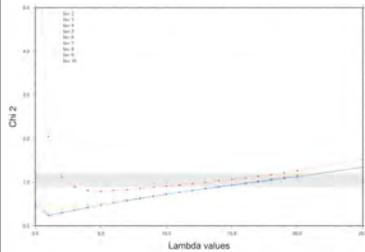
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## FAST - Inverse problem

2 parameter files:  
l.in and i.in to control the inversion.

l.in: parameters for the lambda values behavior  
i.in: parameters for inversion (details in the documentation)



Parametric study for the choice of the lambda:  
- Run at least 10-15 iterations with different values for  $\lambda$ .  
- Check that the  $\chi^2$  is stable for the further iterations  
- Choose the  $\lambda$  which converges to  $\chi^2$  around 1  
-  $\lambda$ : the higher, the better (model smoother)

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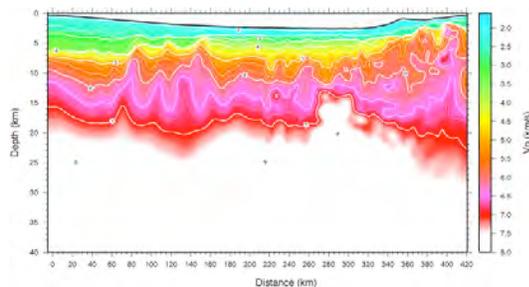
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## FAST - Velocity model



Final model using picks on the MCS data and Chian et al. velocities as a-priori information + layer stripping approach using the option for freezing the upper part of the model.

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### Modeling with TOMO2D (Korenaga et al., 2000)

Model parameterization:

- 2D velocity model parameterized as a sheared mesh; Mesh size variable laterally and with depth; velocity field is made continuous by using bilinear interpolation in each parallelogram-shaped cell; Receivers on the seafloor; Sources anywhere in the model
- Reflector represented by an array of linear segments whose nodal spacing is independent of that used in the velocity grid; Nodes have one degree of freedom (move only vertically); Reflector depth is updated freely without changing adjacent velocity nodes

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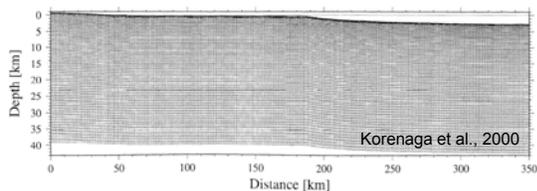
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### Tomo2D - Model parameterization



Mesh size can vary along the model and with depth: allows to have a finer grid near the seafloor and coarser in depth where less structure variations are expected.  
Same mesh for the forward and the inverse problems.

Created using *gen\_smesh* - see documentation for all the options

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## Tomo2D - Forward problem

Forward problem here is the ray tracing.

Need to have it optimized before beginning the inversion process.

Ray-tracing (hybrid) = graph + ray-bending methods

Hybrid travel-time computing is more accurate than with the finite difference method but much more demanding in computing-time.

See "further readings"

Parametric study using *tt\_forward* (parameters in the -N option):

- Run with extreme parameters (huge computing time but the most accurate)
- Find a set the parameters which will give almost the same travel-times with the smallest computing time.

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### Tomo2D - Inverse problem

As the inverse problem is to realize a joint inversion of the first arrivals and the wide-angle reflections arrivals, there are much more parameters to control the inversion with Tomo2D than with FAST.

Parametric study using *tt\_inverse*:

- Use the ray-tracing parameters found with the *tt\_forward* parametric study
- It will actually be parametric studies
- Important parameters to test are (see documentation for definitions): the correlation lengths, the smoothing and the depth kernel weighting factors.
- Other parameter which can be useful: the damping (automatic one tends to be already very good)

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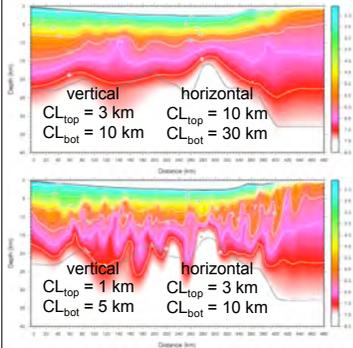
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### Tomo2D - Inverse problem

#### Parametric studies: extremes

#### Correlation lengths for the velocity nodes



Controls the wavelengths of the structures in the model.

Higher CL will lead to smoother model.

Parametric study to find details without over-interpreting the data.

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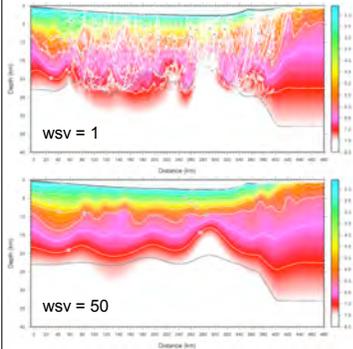
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### Tomo2D - Inverse problem

#### Parametric studies: extremes

#### Weighting factors for the velocity smoothing



Controls the smoothness of the model.

Higher weighting factors will lead to smoother model.

Parametric study to find details without creating artifacts along the ray paths.

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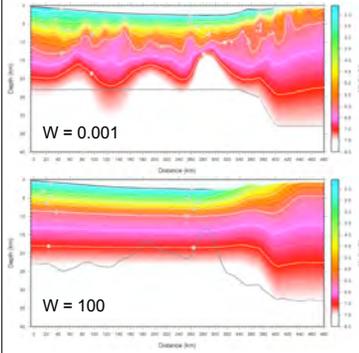
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## Tomo2D - Inverse problem

### Parametric studies: extremes



### Depth kernel weighting factor

Controls the weight of the wide-angle arrival picks compared to the first arrival picks.

A W higher than will tend to fit the wide-angle reflection arrival times in priority.

A W lower than 1 will tend to fit the first arrivals in priority.

Parametric study to find the balance to fit both set of arrivals.

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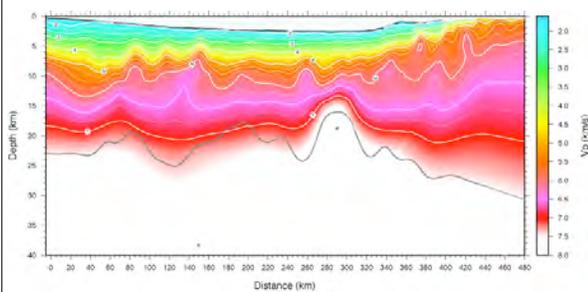
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## Tomo2D - Velocity model



Final model using Chian et al. (2001) and Gerlings et al. (2011) velocities as a-priori information + parametric studies

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### Model's quality and resolution

Statistics:  $\chi^2$ , RMS and N

Constrained areas:

- Masked model,
- Ray-tracing figures,
- DWS

Fits:

- Residuals diagram,
- Fits on the data

Qualitative resolution:

- Checkerboard tests

Quantitative resolution:

- Monte Carlo analysis

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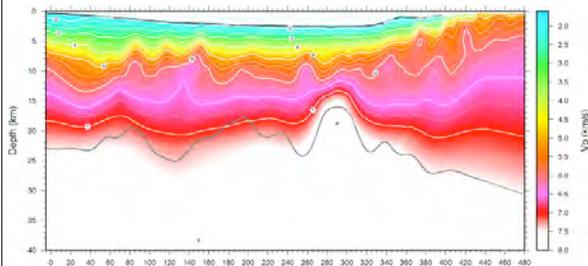
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### Model's quality and resolution

#### Statistics



	$\chi^2$	N	$t_{RMS}$ (s)
1 <sup>st</sup> arrivals	0.88	67,062	0.081
P <sub>m</sub> P	0.79	22,993	0.178
Model	0.86	90,055	0.114

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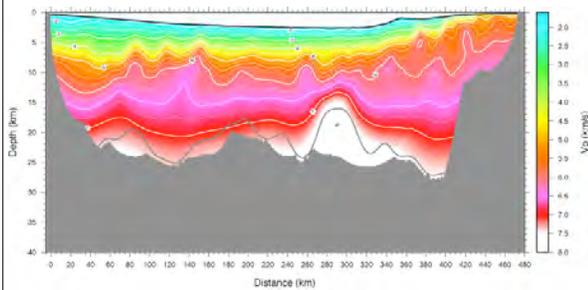
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### Model's quality and resolution

Model masked where there is no ray



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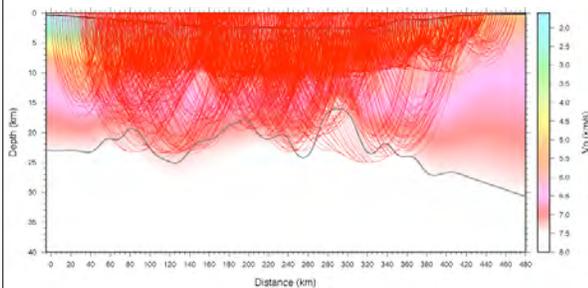
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### Model's quality and resolution

Ray-tracing diagrams:

Refracted rays show where the velocities are sampled



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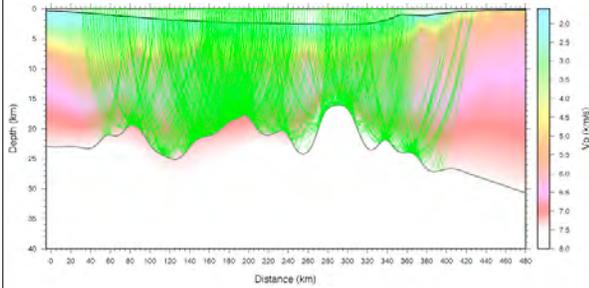
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### Model's quality and resolution

Ray-tracing diagrams:  
Reflected rays show where the interface is sampled



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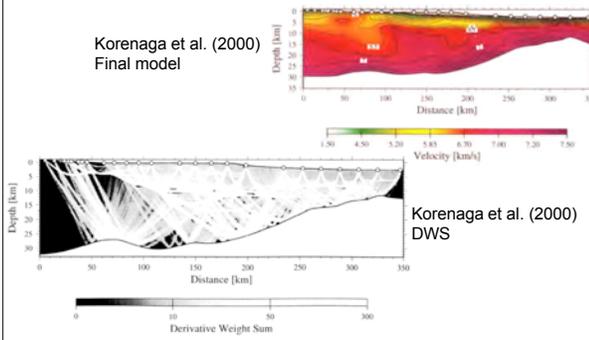
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### Model's quality and resolution

Derivative Weight Sum:  
Another way to show the ray density (velocity sampling) in the model



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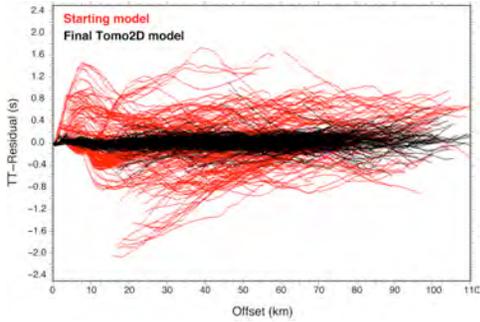
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## Model's quality and resolution

### Residuals



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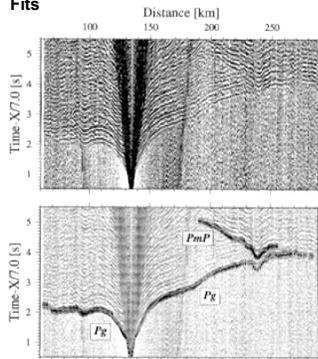
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## Model's quality and resolution

### Fits



Presentation of:  
- The data and only the data  
- The data with the picks (and their uncertainties, as bars) and synthetic traveltimes (as dots)

Korenaga et al. (2000)

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### Model's quality and resolution

#### Checkerboard tests - « How to? »

- Choice of the perturbation pattern (+/- 5 to 10%  $V_p$  following a checkerboard or any other appropriate pattern)
- Perturbate the final model using this pattern
- Compute the traveltimes in this perturbed model
- Add some gaussian randomization to these traveltimes (in the range of the uncertainties)
- Use these perturbed picks as input with the regular initial model
- Compare the output model with the real final model to see where the perturbations are recovered = qualitative resolution

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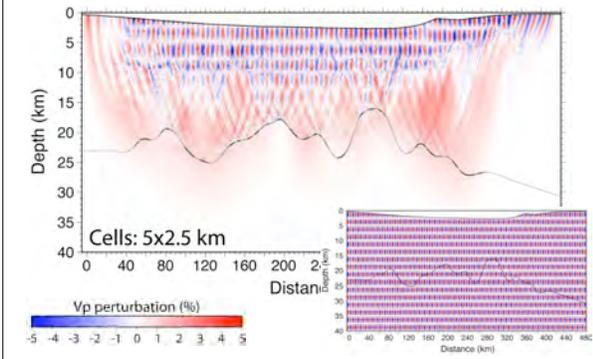
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### Model's quality and resolution

#### Checkerboard tests - example of result



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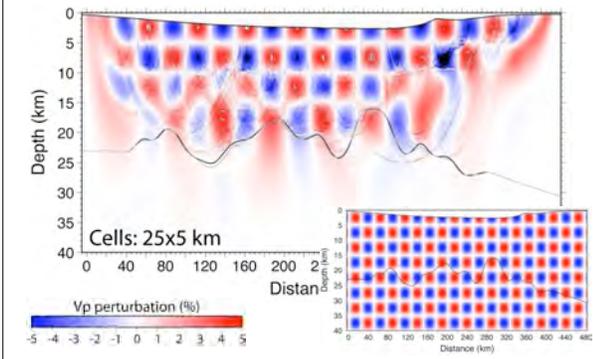
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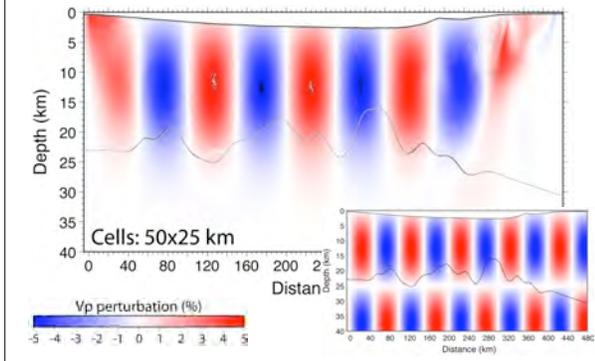
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### Model's quality and resolution

#### Monte Carlo analysis - « How to? »

- Create 100 randomized initial models:
  - Randomization of the velocities (e.g. +/- 5%)
  - Randomization of the depth of the interface (e.g. +/- 3 km for oceanic or thinned continental crust)
- Create a randomized set of data for each initial model:
  - Random picking errors and instrument errors (see "further reading" for details and examples)
- Run the 100 realizations
- Plot average of the 100 output models: should be similar to the preferred model (some modelers use the average as final model)
- Plot the standard deviation of the  $V_p$  and of the depth interface = quantitative resolution

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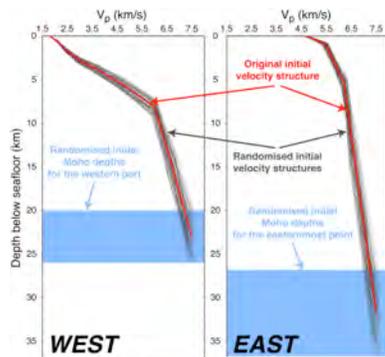
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### Model's quality and resolution

Monte Carlo analysis - 100 randomized input models



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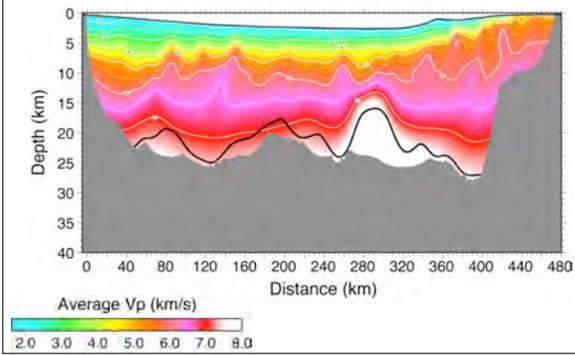
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### Model's quality and resolution

Monte Carlo analysis - average model



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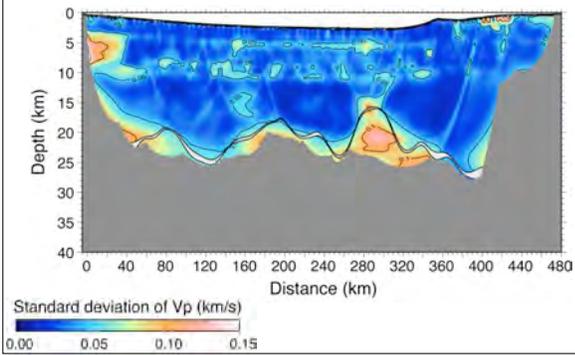
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### Model's quality and resolution

Monte Carlo analysis



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